1 Introduction

They wanted facts. Facts! They demanded facts from him, as if facts could explain anything.

(Joseph Conrad, Lord Jim)

1.1 Motivation

The effect of artificial periodic potential modulations on quasi-two-dimensional carrier systems has raised considerable interest since the realization of this idea in physical devices has been demonstrated by Weiss *et al.* in 1989 [1]. The modulations introduce an additional length scale, the modulation period a, into the system, which is already characterized by several intrinsic dimensions. In the simple case of a nearly free electron gas with effective mass m^* and areal density n_e, these are the Fermi wave length $\lambda_F = 2\pi/k_F$, where $k_F = \sqrt{2\pi n_e}$ is the wave number at the Fermi energy E_F, on the one hand and the mean free path $\ell_f = v_F \tau$, where τ is the relaxation time[1] and $v_F = \sqrt{2E_F/m^*} = \hbar k_F/m^*$ the Fermi velocity, on the other hand. In a magnetic field B_z perpendicular to the plane of the electron sheet, the magnetic length $\ell_m(B_z) = \sqrt{\hbar/eB_z}$ and the cyclotron radius $R_c(B_z) = \ell_m^2 k_F = v_F m^*/eB_z$ also become important. Similar considerations apply to more realistic models.

In high-mobility quasi-two-dimensional electron gases (2DEGs) that appear in real semiconductor structures, the Fermi wave length is typically measured in tens of nanometres,

[1] In general, several distinguishable scattering mechanisms can be relevant.

while the mean free path at liquid helium temperatures can be several microns long. The diameter of the cyclotron orbits of electrons moving at the Fermi velocity lies in this range for moderate magnetic fields of several tenths of a Tesla to several Teslas, which are readily accessible to experiments. The magnetic length ℓ_{m} becomes comparable to a at even smaller fields.

Although the lower end of the scale still poses a significant technological challenge, the creation of superlattices with lattice constants of this size has now become feasible, opening up the possibility of studying physical phenomena that result from the commensurability of ℓ_{m} and R_{c} with the artificially imposed period a. In contrast, while every metallic solid has a periodic potential set up by the atomic nuclei, commensurability effects are not observable since the small inter-atomic distances of a few Ångström and large electron densities mean that a magnetic field of tens of thousands of Teslas would be required to achieve a ℓ_{m} or R_{c} comparable to the lattice spacing [2].

The original motivation for the interest in such systems centred on the commensurability of a and the magnetic length ℓ_{m} and was sparked by Hofstadter's 1976 paper [2]. He pointed out that contemporary theoretical work on the problem implied that the energy spectrum of the electron gas forms a fractal structure, which will be revisited in the next section. Since observation depends on resolving the internal structure of Landau levels, and hence on small thermal and impurity broadening as well as on a strong magnetic field, which implies small a, experimental traces of this intriguing spectrum have been hard to come by and significant progress has only recently been made [3].

After its discovery by Weiss *et al.* [1, 4], the alteration of the transport properties caused by the commensurability of the lattice period and the cyclotron radius has instead been at the centre of attention. Although a quantum mechanical treatment is required to fully explain the observed behaviour, the effect is classical in its origin and results from the distortion of ballistic electron trajectories by the superlattice potential. Depending on the relation between a and R_{c} (and hence B_z), electronic conductivity is enhanced or suppressed; for sufficiently strong modulations—or antidots—a large proportion of the electron orbits becomes chaotic. These phenomena have mostly been investigated in 2DEGs formed in GaAs–$Al_xGa_{1-x}As$ heterostructures and are now quite well understood, although theoretical approaches to strongly modulated systems have largely been limited to numerical simulations. Chaotic electron dynamics in periodic superlattices are still a focus of current research, comprising topics such as directional transport in arrays with broken symmetry connected to an external energy source [5–7].

In InAs–GaSb double heterostructures (DHETS), a 2DEG forms in the InAs layer as a result of charge transfer. Our research group in Oxford has developed metal-organic vapour phase epitaxy (MOVPE) techniques to grow such devices with consistently high concentrations of mobile holes approaching that of the electrons [8, 9]. The motivation behind the research leading to the present thesis has been to study the expression of the semiclassical commensurability phenomena in a more complex system consisting of quasi-two-dimensional gases of both electrons and electron holes. The opportunity to test the generality of previous observations by looking at a different material system and the absence of a depletion layer at the free surface of InAs—which opens up the possibility of smaller effective antidot diameters [10]—have been additional considerations.

1.2 Hofstadter's Butterfly

In the presence of a periodic potential $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{R})$, where $\mathbf{R}$ is a lattice vector, the wave function $\varphi(\mathbf{r})$ of an electron in a magnetic field $\mathbf{B}$ obeys the time-independent Schrödinger equation

$$\hat{H}\varphi(\mathbf{r}) = \frac{[\hat{\mathbf{p}} + e\mathbf{A}(\mathbf{r})]^2}{2m}\varphi(\mathbf{r}) + V(\mathbf{r})\varphi(\mathbf{r}) = E\varphi(\mathbf{r}), \tag{1.1}$$

where $\hat{\mathbf{p}} = -i\hbar\,\nabla_{\mathbf{r}}$ is the momentum operator and $\mathbf{A}(\mathbf{r})$ the magnetic vector potential with $\mathbf{B} = \nabla_{\mathbf{r}}\times\mathbf{A}$. For $\mathbf{A}(\mathbf{r}) = \mathbf{0}$, the eigenstates of $\hat{H}$ are of the Bloch form $\varphi_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r})\exp(i\mathbf{k}\cdot\mathbf{r})$, where $\mathbf{k}$ is the crystal momentum and $u_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R})$; their eigenvalues are the Bloch bands $E_n(\mathbf{k})$ with band index n.

The energy spectrum of Eq. (1.1) may be obtained by solving the Schrödinger-like equation

$$\hat{\hat{H}}\bar{\varphi}(\mathbf{r}) = E\bar{\varphi}(\mathbf{r}), \tag{1.2}$$

where the *effective Hamiltonian* $\hat{\hat{H}}$ to first order in the magnetic field is $E_n(\hat{\boldsymbol{\kappa}})$ with $\hbar\hat{\boldsymbol{\kappa}} = \hat{\mathbf{p}} + e\mathbf{A}(\mathbf{r})$ [11]. For a two-dimensional potential with $\mathbf{R} = a(n_x, n_y)$, where $n_x, n_y \in \mathbb{Z}^+$, the Bloch bands can be approximated by $E_n(k_x, k_y) = E_n^{(0)} + E_n^{(1)}(\cos k_x a + \cos k_y a)$, where $E_n^{(0)}$ and $E_n^{(1)}$ are empirical parameters.[2] Eq. (1.2) then becomes

$$E_n^{(0)}\bar{\varphi}(x,y) + \frac{E_n^{(1)}}{2}\Big[\bar{\varphi}(x+a,y) + \bar{\varphi}(x-a,y) \\ + e^{-ieB_z x/\hbar}\bar{\varphi}(x,y+a) + e^{ieB_z x/\hbar}\bar{\varphi}(x,y-a)\Big] = E\bar{\varphi}(x,y) \tag{1.3}$$

for a single band n, where the Landau gauge $\nabla_{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}) = 0$ with $\mathbf{A}(\mathbf{r}) = (0, B_z x, 0)$ has been used. By defining $n \stackrel{\text{def}}{=} x/a$, $\nu \stackrel{\text{def}}{=} k_y a$, and $\varepsilon \stackrel{\text{def}}{=} 2(E - E_n^{(0)})/E_n^{(1)}$, making the *Ansatz* $\bar{\varphi}(x,y) = \exp(i\nu y/a) g_n$, and introducing the dimensionless parameter $\alpha \stackrel{\text{def}}{=} eB_z a^2/(2\pi\hbar)$, one can simplify Eq. (1.3) to Harper's equation [2, 13]:

$$g_{n+1} + g_{n-1} + 2\cos(2\pi n\alpha - \nu) g_n = \varepsilon g_n. \tag{1.4}$$

The reduced magnetic field $\alpha = (B_z a^2)\big/(h/e) = \Phi/\Phi_0^{(D)}$ is the ratio of the magnetic flux per unit cell $\Phi = B_z a^2$ to the Dirac flux quantum[3] $\Phi_0^{(D)} = h/e$; it can also be expressed in terms of the magnetic length ℓ_{m} as $\alpha = (a/\ell_{\mathrm{m}})^2\big/2\pi$.

The eigenvalue spectrum ε_α of the difference equation (1.4) is the set of all ε for a given α such that for some value of ν there is a g_n (and hence $\bar{\varphi}(x)$ and $\varphi(x)$) that is bounded for all n. It has the peculiar property that it depends on the *rationality* of α, *i.e.*, whether $\alpha = p/q$ for some prime numbers p and q. In this case, there are q distinct energy bands, whereas for irrational α the spectrum consists of infinitely many isolated values. In his famous 1976 paper [2], Hofstadter points out that the union of all ε_α (see Fig. 1.1) forms a self-similar fractal set [14], which has since become known as Hofstadter's butterfly. Its graph is periodic

[2] See, *e.g.*, Kittel's book [12] for a derivation of $E_n(\mathbf{k})$ in the Kronig-Penney framework, from which this semiempirical 'Harper model' may be justified.

[3] Not to be confused with the superconducting flux quantum $\Phi_0^{(s)} = h/2e$.

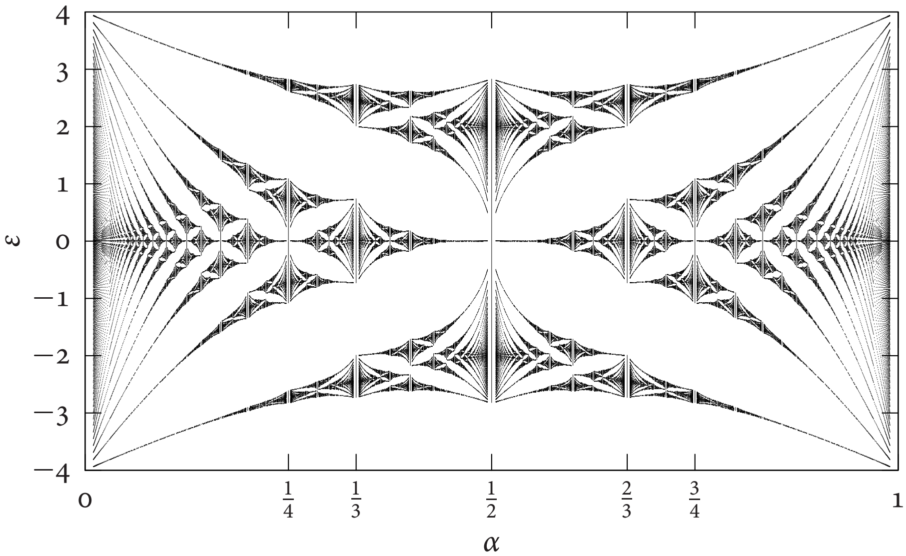

Figure 1.1: Hofstadter's butterfly

in α with period 1—so that the pattern repeats for each additional flux quantum per unit cell—and the unit interval $[0, 1)$ exhibits reflection symmetry in the lines $\varepsilon_\alpha = 0$ and $\alpha = \frac{1}{2}$. At low α, the bands group into clusters which can be identified with the familiar Landau fan that is predicted in the absence of a periodic potential. As is characteristic of fractals arising from mathematical descriptions of physical systems, the unphysical property of being discontinuous everywhere is a formal consequence of the idealized theoretical representation. If a finite uncertainty in B_z is introduced [2], or if a finite size system is considered [15], continuity is recovered, and the scale at which the fractal is resolved depends on the particular model.

Interestingly, Harper's equation (1.4) *also* emerges from first-order perturbation theory in the complementary limit of a weak potential modulation of the form $V(x, y) = V_0(\cos Kx + \cos Ky)$, with $K \overset{\text{def}}{=} 2\pi/a$, perturbing a Landau quantized system [16, 17].[4] In this case, the parameters $\alpha \overset{\text{def}}{=} \Phi_0^{(D)}/\Phi$ and $\nu \overset{\text{def}}{=} -k_y \ell_{\text{m}}^2/(2\pi a)$ have a different meaning—so that the energy spectrum is periodic in $1/B_z$—and Hofstadter's butterfly describes the internal structure of a single Landau level. Hofstadter [2] points out that such a mapping is consistent with

[4] Landau quantization is discussed in Sec. 5.6.2

the scaling properties of the complete pattern. However, the range of validity is different for both approaches and the exact reappearance of the same equation is an artefact of the peculiar choices for $E(k_x, k_y)$ and $V(x, y)$.

1.3 Development

The initial work on the InAs–GaSb DHETs was done using the electron beam lithography (EBL) facilities available to our group in Oxford. These consisted of a converted scanning electron microscope (SEM) with a resolution not much better than 0·5 μm, and it became quickly clear that the lithographical performance of this setup was severely limiting the experiments that could be accomplished. This was especially true as the high density of the required antidot patterns implied a significant contrast reduction due to exposure from secondary electrons. Additionally, the poor selectivity of the applicable etch chemistries for the semiconductor materials over EBL resists meant that the success rate at the intended etch depths was low and could only be improved by switching to a substantially more complicated technique using an intermediate mask.

As the upgrade of the EBL equipment accessible to me in Oxford or through our collaborators was delayed, and a new atomic force microscope (AFM) had become available to our group that could be modified for such work, I began to investigate the option of patterning the DHETs by local anodic oxidation (LAO) using this instrument. While only shallow surface modifications were possible with this approach, preliminary experiments had shown that removal of only a part of the thin GaSb layer forming the uppermost part of the DHETs was enough to impose a significant potential on the carrier gases. Moreover, the technique promised a much improved resolution and greater flexibility in device fabrication.

After LAO performance, which is mostly limited by poor reproducibility, had been optimized to the point where patterned DHETs as originally envisaged could be created, it became apparent that the initial results had been misleading. Under a wide range of conditions, the modulation potential set up by the surface corrugation was insufficient to prompt a measurable effect. In response to this, I modified the technique to allow for deeper modifications by patterning an intermediate etch mask instead of the semiconductor surface. In the meantime, improved EBL facilities had become available in the Cavendish Laboratory in Cambridge and thanks to the help of Geb Jones these could be used to create very similar samples, which formed the basis of much of the work reported in Chapter 7.

Throughout the evolution of this research I worked closely with Nigel Mason and Philip Shields on the growth of InAs–GaSb DHETs by MOVPE in the now-defunct atmospheric pressure reactor in the Clarendon Laboratory. In the context of this work, I undertook measurements on a large number of DHETs both for growth assessment and to improve our understanding of the fundamental properties of these systems, including for the first time a systematic investigation of the magnetotransport properties of structures containing two InAs wells.

1.4 Thesis Structure

The remainder of this thesis can roughly be divided in two parts: Chapters 2, 3, and 4 focus on the engineering aspects, especially the work on lithographical techniques using a scanning probe microscope, while Chapters 5, 6, and 7 are concerned with the physics of InAs–GaSb double heterostructures, paying special attention to lateral potential modulations in the presence of a perpendicular magnetic field. Each part starts with a chapter that sets the scene by providing the essential background information.

In the case of the first part, this is Chapter 2, which describes the operation and the limitations of the atomic force microscope and outlines how the special capabilities of this instrument can be applied to surface manipulation. As the issues are quite general, and to allow for the straightforward comparison with similar lithographic approaches, I shall approach the topic from the point of view of generic scanning probe microscopes and introduce the AFM as an important application of these ideas. Chapter 3 builds on this basis in discussing my work on the modification of GaSb, InAs, and Al surfaces by local anodization with an

AFM. How this technique may be integrated in a fabrication scheme for antidot samples or similar devices is explained in Chapter 4, which also covers the details of the experimental procedures used in the present work.

The second part starts with a an overview of the basic physics of InAs–GaSb DHETs in Chapter 5. In the same chapter, the growth of these structures in our research group in Oxford is surveyed, and the individual DHETs studied in the remainder of the thesis are introduced. Chapter 6 reports studies of a series of substrates containing double InAs wells, relating them to previous work on superlattices. Lateral antidot lattices modulating InAs–GaSb DHETs are finally investigated in Chapter 7, exploring the possibility of a contribution to the observed commensurability effects from the presence of a mobile hole layer.

Chapter 8 concludes the thesis with a summary of the most important results and attempts to assess the lessons learnt in the conduct of this research. What follows is a series of appendices containing material that was excluded from the above chapters so as not to distract from the main exposition, but which I consider none the less important for substantiating some of the claims made in the main text.

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